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Notes on Tachyon Effective Actions and Veneziano Amplitudes

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In a previous paper (hep-th/0304045) it has been argued that tachyonic Dirac-Born-Infeld (DBI) actions can be obtained from open string theory in a limit, which generalizes the usual massless DBI limit. In the present note we review this construction focusing on a key property of the proposed tachyon effective actions: how they reproduce appropriate Veneziano amplitudes in a suitably defined kinematical region. Possible extensions and interesting open problems are briefly discussed.

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1. Introduction

The appearance of a tachyon in the perturbative spectrum of string theory (open or closed) signals the presence of an instability that drives the theory away from the unstable vacuum. During this time-dependent process the tachyon mode grows exponentially and couples, in general, to all the other modes of the string. The spacetime dynamics of this process is captured by a nontrivial string field theory action, whose formulation and analysis is a complicated problem. In this note we argue, following [1], that in certain limits in the configuration space of string theory the tachyon decouples from the other massive stringy modes. In such cases the spacetime dynamics of the tachyon is expected to be captured by a much simpler effective action, whose formulation is the main subject of this paper. This effective action is useful for various purposes. It contributes to a further understanding of the time-dependent dynamics of tachyon condensation in string theory and provides a useful tool for interesting applications in more realistic setups including recent discussions in cosmology (see, for example, [2] and references therein).

To motivate our discussion, let us recall a more familiar case: the massless Dirac-Born-Infeld (DBI) effective action in open string theory [3]. For our purposes it is enough to consider the ten-dimensional Born-Infeld Lagrangian

$$\mathcal{L}_{\rm BI}=\sqrt{-\det(\eta_{\mu\nu}+2\pi\alpha' F_{\mu\nu})}\ , \tag{1.1}$$

describing the dynamics of a $U(1)$ gauge field A_μ on a single D-brane. Some of the key properties of this Lagrangian in the bosonic case are the following:

(1) It has a surface of exact solutions parameterized by constant $F_{\mu\nu}$ profiles. These are also exact solutions of the full open string equations of motion, i.e. the massless vertex operator $F_{\mu\nu}X^\mu\partial X^\nu$ is a true modulus of the theory.

(2) As an effective Lagrangian (1.1) is exact for arbitrary constant values of $F_{\mu\nu}$ (the only restriction coming from a critical upper bound on the value of the electric field).

(3) In string theory the general scattering amplitude of n gauge bosons with momenta p_j and polarizations ζ_{μ_j} (respecting the standard string BRST conditions) takes the form

$$\begin{aligned}\mathcal{I}_n(\{\zeta_{\mu_j}\},\{p_j\})=\int_{-\infty}^{\infty}\prod_{i=1}^{n-3}dy_i\ \prod_{j=1}^{n}\zeta_{\mu_j}\langle c\partial X^{\mu_1}e^{ip_1X}(0)c\partial X^{\mu_2}e^{ip_2X}(1)\times\\ \times\, c\partial X^{\mu_3}e^{ip_3X}(\infty)\partial X^{\mu_4}e^{ip_4X}(y_1)\cdots\rangle\ +\ \text{inequivalent orderings.}\end{aligned} \tag{1.2}$$

In this expression c denotes the usual bosonic string ghost. To leading order in the external momenta p_i the n-point function (1.2) scales like $(\text{momentum})^n$. The same scattering amplitudes can be computed from the Born-Infeld Lagrangian. They receive two types of contributions: one comes from the one-particle-irreducible (1PI) diagrams at order n and the other from reducible exchange diagrams involving lower order irreducible vertices connected by propagators. To leading order in momentum these n-point functions again scale like $(\text{momentum})^n$. To order $(\text{momentum})^n$, the string theory amplitudes (1.2) agree with those obtained from the Born-Infeld action. An explicit check of this claim up to 4th order can be found in [4].

In fact, this is one way to determine the form of the Born-Infeld action. We can write down the most general gauge-invariant local Lagrangian containing all possible independent invariants of increasing dimensions and the properties (1)-(3) will fix the form of this Lagrangian up to field redefinitions [4].

An alternative way to summarize the above set of properties is the following. In the σ-model approach to string theory (see, for example, [5,6]) the configuration space of the theory can be viewed as the space of (in general non-conformal) two-dimensional worldsheet quantum field theories. The constant $F_{\mu\nu}$ profiles parameterize a surface of fixed points of the worldsheet renormalization group (RG) and the Born-Infeld effective action governs the dynamics of small (low-energy) fluctuations away from this surface of fixed points.

Is it possible to find a suitable extension of this construction that incorporates the dynamics of the open string tachyon? The massless DBI example suggests the following course of action. First, we have to find a line of fixed points of the worldsheet RG, i.e. tachyon profiles that are exactly marginal. For example, we may consider the homogeneous rolling tachyon solution of Sen [7] in open superstring theory[1]

$$T = T_+ e^{\frac{1}{\sqrt{2}} x^0} + T_- e^{-\frac{1}{\sqrt{2}} x^0} \ . \tag{1.3}$$

This is parameterized by the arbitrary constants $T_\pm$ [2] and is known to be an *exact* solution of the full open string equations of motion in the Euclidean signature [9,10,11]. The same is

[1] In this paper we discuss only the case of unstable non-BPS D-branes in type II superstring theory. For relevant details on the bosonic case see [1,8].

[2] Strictly speaking, inequivalent solutions are parameterized by only one parameter. This is due to the freedom of shifting x^0 by an arbitrary constant.

believed to be true also in the Minkowki theory after the analytic continuation $x^0 \to ix^0$. The precise problem we want to solve is the following. For starters, consider type II superstring theory in flat space, i.e. in the absence of a rolling tachyon condensate. We can write the following open string tachyon vertex operators

$$T^{(\pm)}_{\vec{k}} = e^{w_\pm x^0 + i\vec{k}\cdot\vec{x}} \ . \tag{1.4}$$

These vertex operators are in the (-1)-picture and the on-shell condition reads[3]

$$w_\pm^2 + \vec{k}^2 = \frac{1}{2} \ , \tag{1.5}$$

with the $\pm$ indices denoting the two opposite sign solutions of this quadratic equation. $\vec{k} = 0$ for the rolling tachyon vertex operators $e^{\pm\frac{1}{\sqrt{2}}x^0}$ and small fluctuations around them are given by small *spatial* momenta and frequencies

$$w_\pm = \pm\frac{1}{\sqrt{2}}(1 - \vec{k}^2) + O(\vec{k}^4). \tag{1.6}$$

We want to study the leading order behaviour of the n-point Veneziano amplitudes

$$\mathcal{A}_n = \langle T^{(\pm)}_{\vec{k}_1} T^{(\pm)}_{\vec{k}_2} T^{(\pm)}_{\vec{k}_3} \cdots T^{(\pm)}_{\vec{k}_n} \rangle \tag{1.7}$$

in the limit of small *spatial* momentum. In the process of taking this limit we have broken the explicit 10-dimensional Poincare invariance, but this breaking is obviously spontaneous. The choice of the time direction is arbitrary and the final expression of the amplitudes $\mathcal{A}_n$ will be Poincare invariant. In section 2 we find the following structure. The amplitudes $\mathcal{A}_n$ vanish automatically for n odd and for n even they are certain Lorentz invariant functions of the external momenta, which scale like (momentum)^2 to leading order in spatial momenta. These quadratic functions can be split into two distinct pieces. The first one is a quadratic polynomial P_n of appropriate Mandelstam variables. The second is a rational expression of the spatial momenta - let us call it W_n - that incorporates the complicated pole structure of $\mathcal{A}_n$ arising from intermediate on-shell tachyons. There are no massless or higher mass stringy state poles. The higher mass stringy states are automatically decoupled in the special kinematics of small spatial momentum. The form of P_n is completely fixed by

[3] In our conventions $\alpha' = 1$.

symmetry considerations and it has *one* free coefficient at each order n. W_n arises as a sum of lower order exchange diagrams and therefore is fixed by induction.[4]

We would like to obtain an action with the following well-defined properties:

(a) The equations of motion of this action admit the rolling tachyon solutions (1.3) as exact solutions for any constants $T_{\pm}$.

(b) The field theory amplitudes (1.7) obtained from this action reproduce the corresponding string theory Veneziano amplitudes to the leading quadratic order in spatial momenta, i.e. they have the leading order structure described in the previous paragraph.

If this action exists it will not be unique. String theory amplitudes are, by definition, on-shell and there are well-known ambiguities in trying to determine an effective Lagrangian purely from on-shell data. One has the freedom to change its form performing field redefinitions $T \to f(T, \partial_\mu T, \partial_\mu \partial_\nu T, ...)$ and/or adding couplings that are proportional to the equations of motion. In section 3 these ambiguities will be fixed in a very specific way. We consider an effective Lagrangian of the form

$$\mathcal{L} = \mathcal{L}(T, \partial_\mu T) \tag{1.8}$$

and demonstrate that this (single-derivative) ansatz is capable of satisfying both requirements (a) and (b). Imposing condition (a) fixes the form of the Lagrangian (1.8) uniquely up to *one* free coefficient at each order in T. Let us call this coefficient a_n. Then, we compute the leading order form of the amplitudes (1.7) in field theory and we find that they have the same quadratic structure as in string theory. In particular, they contain *one* free parameter at each order n and this parameter, which is a simple function of a_n, is fixed by imposing requirement (b). This procedure determines $\mathcal{L}$ completely up to field redefinitions. As usual, different and more complicated actions with the same properties can be obtained in different schemes.

So far we have considered string theory in flat space. We would like to consider the extension of the above analysis in the presence of non-vanishing tachyon condensates. In

[4] Compare this situation with the corresponding one in the usual massless DBI case described above. In that case the n-point functions scale like $(\text{momentum})^n$ to leading order in momenta and receive polynomial contributions from 1PI diagrams and a complicated pole structure from exchange diagrams. In the tachyon case, the n-point amplitudes scale like $(\text{spatial momentum})^2$ to leading order in spatial momenta and receive a similar type of contributions summarized in the functions P_n, W_n above.

section 4 we argue that if we can treat the theory perturbatively in T_+, the same action will continue to reproduce the leading quadratic terms of the perturbed string theory scattering amplitudes

$$\langle T^{(\pm)}_{\vec{k}_1} T^{(\pm)}_{\vec{k}_2} \cdots T^{(\pm)}_{\vec{k}_n} \rangle_{T_+} \tag{1.9}$$

in the presence of a half-brane rolling tachyon solution (T_+ arbitrary and $T_- = 0$ in (1.3)). This extends the validity of this action in the presence of a non-vanishing T_+-condensate. When we try to extend the same reasoning in the full-brane case (both T_+ and T_- non-vanishing in (1.3)), we encounter several complications and the action (1.8) breaks down. The properties of an alternative effective action are briefly discussed.

We would like to point out that the tachyon effective action obtained in the above manner is a special case of a more general construction with the following key characteristics. Around a line (or surface) of fixed points in the configuration space of string theory small fluctuations of the tachyon and/or massless modes decouple from the other massive modes of the string. This decoupling limit generalizes the low-energy limit of the massless case in an interesting new way, in which the excited fields fluctuate infinitesimally around a line of classical solutions. In this note we argue in favor of an effective action that describes correctly the interactions of these small fluctuations. We do so in a special setup in open string theory. Possible extensions to other cases and interesting open problems are suggested in section 5.

2. String theory Veneziano amplitudes with vanishing tachyon condensates

To calculate the Veneziano amplitudes $\mathcal{A}_n$ in (1.7) it is convenient to Wick rotate $x^0 \to ix^0$ and consider the vertex operators (1.4) in Euclidean signature. Then, in the vanishing spatial-momentum limit the correlation functions (1.7) involve momentum modes, whose momentum vectors are almost aligned with a particular, arbitrarily chosen, axis in Euclidean space. We perform the computation of $\mathcal{A}_n$ in this special kinematical region in Euclidean signature and at the end we continue back to Minkowski space. At the current level of understanding of string theory this is the standard way to compute open string scattering amplitudes.

Odd-point functions will vanish automatically by momentum conservation. On the other hand, non-vanishing $2n$-point functions involving the positive frequency momenta

$\{k_1, ..., k_n\}$ and the negative frequency momenta $\{p_1, ..., p_n\}$ have to satisfy the momentum conservation constraints[5]

$$\begin{aligned} &\sum_{i=1}^{n} (\vec{k}_i + \vec{p}_i) = 0 \ , \\ &\sum_{i=1}^{n} \vec{k}_i^2 = \sum_{i=1}^{n} \vec{p}_i^2 \ . \end{aligned} \tag{2.1}$$

They can be computed on the upper-half plane in the usual manner by fixing the position of three vertex operators and integrating over the rest:

$$\begin{aligned} \mathcal{A}_{2n}(\{\vec{k}_i\}, \{\vec{p}_j\}) = g_o^{2n-2} \mathcal{C} \int_{-\infty}^{\infty} \prod_{i=1}^{2n-3} dy_i \langle T_{\vec{k}_1}^{(+)}(0) T_{\vec{k}_2}^{(+)}(1) \mathcal{T}_{\vec{k}_3}^{(+)}(\infty) \times \\ \times \mathcal{T}_{\vec{k}_4}^{(+)}(y_1) \cdots \mathcal{T}_{\vec{k}_n}^{(+)}(y_{n-3}) \mathcal{T}_{\vec{p}_1}^{(-)}(y_{n-2}) \cdots \mathcal{T}_{\vec{p}_n}^{(-)}(y_{2n-3}) \rangle + (k_2 \leftrightarrow k_3) \ . \end{aligned} \tag{2.2}$$

As usual, the total picture number in superstring amplitudes must be equal to -2. In (2.2) we have inserted two vertex operators in the (-1)-picture and the rest in the 0-picture. In this paper we reserve the notation $\mathcal{T}_{\vec{k}}^{(\pm)}$ for the 0-picture vertex operators

$$\mathcal{T}_{\vec{k}}^{(\pm)} = i(\vec{k} \cdot \vec{\psi} - w_{\pm} \psi^0) e^{i w_{\pm} x^0 + i \vec{k} \cdot \vec{x}} \ . \tag{2.3}$$

g_o is the open string coupling and $\mathcal{C}$ a universal constant that can be determined by unitarity. For example, inspecting the way that a 4-point function factorizes into the product of two 3-point functions near a massless pole we obtain $\mathcal{C} = 1$ [12].

The leading order form of these amplitudes can be reduced considerably with a few simple observations. First of all, when we set the spatial momenta $\{\vec{k}_i\}$, $\{\vec{p}_j\}$ to zero, the amplitudes vanish as a simple manifestation of the fact that the rolling tachyon vertex operators $e^{\pm \frac{i}{\sqrt{2}} x^0}$ are exactly marginal. This is a special case of a more general statement. Correlation functions of true moduli of string theory are identically zero. An interesting exception to this rule, relevant for the massless DBI case, is the following. Consider the n-point vector amplitudes (1.2) in bosonic string theory (the situation in fermionic string theory is similar). The first non-trivial term in the momentum expansion of the gauge boson vertex operators $\mathcal{V}_{\zeta,p} = \zeta_\mu \partial X^\mu e^{ip\cdot X}$ has the form $\zeta_\mu p_\nu X^\nu \partial X^\mu$ and the amplitude

$$\begin{aligned} \mathcal{I}_n^{(0)}(\{\zeta_{\mu_j}\}, \{p_j\}) = \int_{-\infty}^{\infty} \prod_{i=1}^{n-3} dy_i \ \prod_{j=1}^{n} \zeta_{\mu_j} \langle c p_{\nu_1} X^{\nu_1} \partial X^{\mu_1}(0) c p_{\nu_2} X^{\nu_2} \partial X^{\mu_2}(1) \times \\ \times c p_{\nu_3} X^{\nu_3} \partial X^{\mu_3}(\infty) p_{\nu_4} X^{\nu_4} \partial X^{\mu_4}(y_1) \cdots \rangle \ + \ \text{inequivalent orderings} \end{aligned} \tag{2.4}$$

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[5] Henceforth this will be standard notation. Momenta with a positive frequency will be denoted by k_i and momenta with a negative frequency will be denoted by p_i.

gives the leading contribution to the full vector amplitude (1.2) in the limit of small momenta. Indeed, this is one way to see that the leading term in (1.2) scales like (momentum)^n as we claimed above. The symmetric part of the vertex operator $\zeta_\mu p_\nu X^\nu \partial X^\mu$ is a trivial total derivative and the antisymmetric part has the form $f_{\mu\nu} X^\nu \partial X^\mu$, with $f_{\mu\nu}$ a constant antisymmetric tensor. This is the familiar coupling of a constant electromagnetic field on the boundary of the disk. It is a true modulus of open string theory and the above general statement implies that the corresponding amplitudes (2.4) have to vanish. An explicit calculation, however, demonstrates that this conclusion is incorrect. As we might expect also from the massless DBI effective action, the amplitudes (2.4) are non-vanishing. The reason for this, perhaps unexpected, result is the fact the vertex operators $\zeta_\mu p_\nu X^\nu \partial X^\mu$ are not well-defined CFT vertex operators and the computation of the corresponding amplitudes is subtle.

In the case of the tachyon no subtleties of the above type appear and the leading order form of the amplitudes (2.2) is quadratic. This quadratic dependence can be written as a function of the form

$$\mathcal{A}_{2n}(\{\vec{k}_i, \vec{p}_j\}) = g_o^{2n-2}\big(P_{2n}(\vec{k}_i, \vec{p}_j) + W_{2n}(\vec{k}_i, \vec{p}_j)\big) \ . \tag{2.5}$$

$P_{2n}(\vec{k}_i, \vec{p}_j)$ is a second order polynomial of the spatial momenta and $W_{2n}(\vec{k}_i, \vec{p}_j)$ a second order rational expression of the momenta with a sum of terms, each scaling like (momentum)$^{2s+2}$/(momentum)^{2s} (for appropriate term-dependent positive integers s). The denominators of these terms are products of propagators associated to intermediate nearly on-shell tachyons and the numerators are determined uniquely as appropriate residues. A particular example of this structure will be given below at 6th order. The higher order pole structure will also be discussed.

The structure of the polynomial P_{2n}, on the other hand, is particularly simple. It is straightforward to determine its exact form at any order $2n$ up to a constant multiplicative factor by invoking the symmetry properties of the amplitudes $\mathcal{A}_{2n}$. This can be done most conveniently with the use of the Mandelstam variables

$$\begin{aligned}
S_{ij} &= (k_i + k_j)^2 \sim 2 - (\vec{k}_i - \vec{k}_j)^2 = 2 - s_{ij} \ , \\
T_{ij} &= (p_i + p_j)^2 \sim 2 - (\vec{p}_i - \vec{p}_j)^2 = 2 - t_{ij} \ , \quad i, j = 1, ..., n, \\
U_{ii'} &= (k_i + p_{i'})^2 \sim (\vec{k}_i + \vec{p}_{i'})^2 = u_{ii'} \ , \quad i, i' = 1, ..., n \ ,
\end{aligned} \tag{2.6}$$

of which only a subset are linearly independent variables. We have introduced the small letter symbols s_{ij}, t_{ij} and $u_{ii'}$, to denote particular quadratic combinations of the spatial

momenta, which will be referred to as the "*spatial* Mandelstam variables". As we send $\{\vec{k}_i\}$, $\{\vec{p}_j\}$ to zero, the Mandelstam variables take the limiting values:

$$S_{ij}, T_{ij} \to 2, \quad U_{ii'} = u_{ii'} \to 0, \quad s_{ij}, t_{ij} \to 0 \ . \tag{2.7}$$

With the use of these variables, P_{2n} can be cast into the Euclidean invariant form

$$P_{2n} = D + \sum_{i,j} (A_{ij} S_{ij} + B_{ij} T_{ij}) + \sum_{i,i'} C_{ii'} U_{ii'} \ . \tag{2.8}$$

$A_{ij}, B_{ij}, C_{ii'}$ and D are appropriate constants, which can be constrained further by the following symmetries of the amplitudes (2.2). First, there is a symmetry under the exchange of the $\{k_i\}$ momenta among themselves and similarly under the exchange of the $\{p_j\}$ momenta among themselves. This property equates all the coefficients A_{ij}, B_{ij} and $C_{ii'}$ to the constants A, B and C respectively. Moreover, the amplitudes (2.2) are invariant under the transformation $(x^0, \vec{x}) \to (-x^0, -\vec{x})$, which exchanges the $\{k_i\}$ momenta with the $\{p_j\}$ momenta. This symmetry sets $A = B$ and leads to the form

$$P_{2n} = D + A \sum_{i,j} (S_{ij} + T_{ij}) + C \sum_{i,i'} U_{ii'} \ . \tag{2.9}$$

One can easily check the identity

$$\sum_{i,j} (S_{ij} + T_{ij}) = 4n^2 - 2 \sum_{i,i'} U_{ii'} \ , \tag{2.10}$$

which allows for a further simplification

$$P_{2n} = (D - 4n^2 A) + (C - 2A) \sum_{i,i'} U_{ii'} \ . \tag{2.11}$$

As we mentioned above, the zeroth order term vanishes. This sets $D = 4n^2 A$ and after denoting the constant $C - 2A$ by C_{2n} we are left with the unique (up to a multiplicative constant) expression

$$P_{2n} = C_{2n} \sum_{i,i'} U_{ii'} = C_{2n} \sum_{i,i'} u_{ii'} \ . \tag{2.12}$$

We proceed to verify and elucidate the general structure of eqs. (2.5), (2.12) with more explicit calculations - first at the lower 4th and 6th orders and then inductively at the higher ones.

2.1. 4-point Veneziano amplitudes

Setting $n = 2$ in the general amplitude (2.2) gives

$$\begin{aligned}\mathcal{A}_4 &= g_o^2 \int_{-\infty}^{\infty} dy \langle e^{ik_1\cdot x}(0) e^{ik_2\cdot x}(1)(p_1\cdot\psi)e^{ip_1\cdot x}(\infty)(p_2\cdot\psi)e^{ip_2\cdot x}(y)\rangle + (k_2 \leftrightarrow p_1) = \\ &= g_o^2(p_1\cdot p_2)\int_{-\infty}^{\infty} dy |y|^{2k_1\cdot p_2}|1-y|^{2k_2\cdot p_2} + (k_2 \leftrightarrow p_1) \ .\end{aligned} \tag{2.13}$$

Further manipulation of this expression produces the following function of the spatial Mandelstam variables

$$\begin{aligned}\mathcal{A}_4 = g_o^2\big[&(1-s_{12})B(u_{12'},u_{11'}) + (-1+u_{11'})B(u_{12'},2-s_{12}) + \\ &+ (-1+u_{12'})B(2-s_{12},u_{11'})\big] \ .\end{aligned} \tag{2.14}$$

In standard notation, $B(s,t)$ denotes the Beta-function

$$B(s,t) = \int_0^1 dy\ y^{-1+s}(1-y)^{-1+t} = \frac{\Gamma(u)\Gamma(t)}{\Gamma(s+t)}. \tag{2.15}$$

(2.14) can be expanded to quadratic order in spatial momenta (i.e. linear order in the spatial Mandelstam variables) using the Γ-function expansion

$$\Gamma(\epsilon) \sim_{\epsilon\to 0} \frac{1}{\epsilon} - \gamma + \frac{1}{2}(\gamma^2 + \zeta(2))\epsilon \ , \tag{2.16}$$

where γ denotes the Euler-Mascheroni constant and $\zeta(2) = \frac{\pi^2}{6}$. At the end of this calculation we find the leading order expression

$$\mathcal{A}_4 = -g_o^2\frac{\pi^2}{4}\sum_{ii'} u_{ii'} \ , \tag{2.17}$$

which is precisely of the general form (2.5), (2.12), with $C_4 = -\frac{\pi^2}{4}$ and $W_4 = 0$. Notice the expected vanishing of the 4-point amplitude at zero spatial momenta.

Another notable characteristic of the 4-point amplitude (2.17) is the absence of any poles. In general, string theory amplitudes possess a complicated pole structure associated to the appearance of on-shell intermediate states. An intermediate state can be a tachyon, a massless excitation or a higher excited stringy state. In our case, the 4th order amplitude does not exhibit any of these poles. It does not exhibit any tachyon poles, because of the absence of a non-vanishing 3-point vertex among tachyons. Massless poles, for example

of the type $\frac{1}{u_{11'}}$ etc., cancel identically after summing over all the channels and massive stringy states only have finite contributions in the special kinematics regime of this note. In higher orders, we expect to find a similar absence of massless and higher mass poles. For the massless poles this is guaranteed only in the abelian case, which is the case of interest in this paper. For the higher mass poles it is a consequence of the special kinematics. The absence of these poles is an important property of the present construction and is equivalent to the statement of the introduction that small fluctuations around the exactly marginal rolling tachyon direction decouple from any other massive excitation of the open string. It is one of the key elements that allows for the possibility of a consistent single-scalar tachyon effective action.

2.2. 6-point Veneziano amplitudes

The 6th order Veneziano amplitude in string theory reads

$$\begin{aligned}\mathcal{A}_6 &= \langle T^{(+)}_{\vec{k}_1} T^{(+)}_{\vec{k}_2} \mathcal{T}^{(+)}_{\vec{k}_3} \mathcal{T}^{(-)}_{\vec{p}_1} \mathcal{T}^{(-)}_{\vec{p}_2} \mathcal{T}^{(-)}_{\vec{p}_3} \rangle = \\ &= \int_{-\infty}^{\infty} dx_1 dx_2 dx_3 \langle e^{ik_1\cdot x}(0) e^{ik_2\cdot x}(1)(k_3\cdot\psi)e^{ik_3\cdot x}(\infty)\times \\ &\times (p_1\cdot\psi)e^{ip_1\cdot x}(x_1)(p_2\cdot\psi)e^{ip_2\cdot x}(x_2)(p_3\cdot\psi)e^{ip_3\cdot x}(x_3)\rangle + (k_1\leftrightarrow k_3)\ .\end{aligned} \tag{2.18}$$

By further evaluation we find

$$\begin{aligned}\mathcal{A}_6 = &(k_3\cdot p_1)(p_2\cdot p_3)\int_{-\infty}^{\infty}\prod_i dx_i |x_1|^{2k_1\cdot p_1}|x_2|^{2k_1\cdot p_2}|x_3|^{2k_1\cdot p_3}\times \\ &\times |1-x_1|^{2k_2\cdot p_1}|1-x_2|^{2k_2\cdot p_2}|1-x_3|^{2k_2\cdot p_3}|x_{12}|^{2p_1\cdot p_2}|x_{13}|^{2p_1\cdot p_3}|x_{23}|^{-1+2p_2\cdot p_3} - \\ &- (k_3\cdot p_2)(p_1\cdot p_3)\int_{-\infty}^{\infty}\prod_i dx_i |x_1|^{2k_1\cdot p_1}|x_2|^{2k_1\cdot p_2}|x_3|^{2k_1\cdot p_3}\times \\ &\times |1-x_1|^{2k_2\cdot p_1}|1-x_2|^{2k_2\cdot p_2}|1-x_3|^{2k_2\cdot p_3}|x_{12}|^{2p_1\cdot p_2}|x_{13}|^{-1+2p_1\cdot p_3}|x_{23}|^{2p_2\cdot p_3} + \\ &+ (k_3\cdot p_3)(p_1\cdot p_2)\int_{-\infty}^{\infty}\prod_i dx_i |x_1|^{2k_1\cdot p_1}|x_2|^{2k_1\cdot p_2}|x_3|^{2k_1\cdot p_3}\times \\ &\times |1-x_1|^{2k_2\cdot p_1}|1-x_2|^{2k_2\cdot p_2}|1-x_3|^{2k_2\cdot p_3}|x_{12}|^{-1+2p_1\cdot p_2}|x_{13}|^{2p_1\cdot p_3}|x_{23}|^{2p_2\cdot p_3} + (k_1\leftrightarrow k_3)\end{aligned} \tag{2.19}$$

with a sum of six terms coming from every possible contraction of the fermions. From the general considerations of the previous subsections the leading order form of this amplitude is expected to become

$$\mathcal{A}_6 = g_o^4\big(C_6\sum_{i,i'} u_{ii'} + W_6(\vec{k}_i,\vec{p}_j)\big)\ . \tag{2.20}$$

Unfortunately, we are not aware of a closed expression for the above integrals and this hinders the computation of the exact value of the constant C_6. Nevertheless, it is quite straightforward to determine the precise form of the rational function W_6. This function involves a set of poles, which are associated to on-shell intermediate tachyons. To be concrete, let us consider a particular tachyon pole, say the one arising when an intermediate tachyon of momentum $k_1 + k_2 + p_1$ goes on-shell. In this limit, the 6-point amplitude factorizes into the product of two lower order 4-point amplitudes and the following expression is obtained

$$\mathcal{A}_6(k_i, p_j) \sim \frac{\mathcal{A}_4(k_1, k_2, p_1, -k_1 - k_2 - p_1)\mathcal{A}_4(-k_3 - p_2 - p_3, k_3, p_2, p_3)}{(k_1 + k_2 + p_1)^2 - \frac{1}{2}} \ . \tag{2.21}$$

By using the leading order expression (2.17) we find

$$\mathcal{A}_6(k_i, p_j) \sim \frac{\pi^4}{4} g_o^4 \frac{s_{12} t_{23}}{(k_1 + k_2 + p_1)^2 - \frac{1}{2}} \ . \tag{2.22}$$

Summing over the nine different contributions of this type we obtain the full dependence of the function W_6 on the spatial momenta $\{\vec{k}_i\}$, $\{\vec{p}_j\}$.

2.3. The pole structure of 2n-point Veneziano amplitudes

Based on the above analysis (see eqs. (2.5) and (2.12)), the general $2n$-point Veneziano amplitude (2.2) can be written to leading order as

$$\mathcal{A}_{2n}(\vec{k}_i, \vec{p}_j) = C_{2n} \sum_{i,i'} u_{ii'} + W_{2n}(\vec{k}_i, \vec{p}_j) \ . \tag{2.23}$$

The g_o dependence has been included implicitly into the definition of the constant C_{2n} and the function W_{2n}. This function incorporates the complicated pole structure of the full amplitude (2.2) in the special kinematical regime of this note. The general characteristics of W_{2n} are the following. There are many channels in (2.2), where poles appear. They arise from regions of the moduli integrals in (2.2), where some number of the y_i variables approach each other. For example, to analyze the limit where $2N$ of them approach 0, e.g. $y_1, y_2, ..., y_N, y_{n-2}, ..., y_{n-N-1} \to 0$, it is convenient to redefine

$$y_1 = \epsilon, \ y_2 = \epsilon z_2, ..., \ y_N = \epsilon z_N, \ y_{n-2} = \epsilon z_{n-2}, ..., \ y_{n-N-1} = \epsilon z_{n-N-1} \tag{2.24}$$

and consider the limit $\epsilon \to 0$ [13]. The residues of the resulting poles are related to lower order correlation functions of intermediate states. Only poles associated to an intermediate

tachyon are interesting. Other types of poles do not appear for the reasons presented above. For concreteness, let us consider the channel (2.24), where an intermediate tachyon of momentum $P = k_1 + k_4 + ... + k_{N+3} + p_1 + ... + p_N$ goes on-shell. Near this pole (2.2) becomes

$$\mathcal{A}_{2n} \sim \frac{\mathcal{A}_{2N+2}(k_1, k_4, ..., k_{N+3}, p_1, ..., p_N, P)\mathcal{A}_{2(n-N)}(-P, k_{N+4}, ..., k_n, p_{N+1}, ..., p_n)}{P^2 - \frac{1}{2}} , \tag{2.25}$$

which is a higher order analogue of (2.21). The form of the lower order amplitudes to leading (quadratic) order is known by induction. The full structure of W_{2n} arises as a sum of similar contributions over all possible channels. In this sum multiple poles scale as $(\text{momentum})^{2s+2}/(\text{momentum})^{2s}$ for appropriate term-dependent positive integers s.

3. On-shell amplitudes in field theory and comparison with string theory

In field theory we look for an effective tachyon Lagrangian with enough parameters to reproduce the string theory scattering amplitudes (2.23). It has been proposed [1] that the single derivative ansatz

$$\mathcal{L} = \mathcal{L}(T, \partial_\mu T) , \tag{3.1}$$

symmetric under the parity transformation $T \to -T$, is a convenient and consistent ansatz that has enough parameters to encode the full set of string theory information to leading order. The even parity condition is an immediate consequence of the standard Z_2 symmetry $\psi_\mu \to -\psi_\mu$ of the fermionic string, under which the open string tachyon is odd. The single derivative Lagrangian (3.1) fixes part of the field redefinition ambiguity and leaves a residual freedom of taking

$$T \to T f(T^2) \tag{3.2}$$

for an arbitrary function f.

In addition, assume that $\mathcal{L}$ is analytic in T around $T = 0$. This assumption is known to fail in the bosonic case [1], but no problems of this sort are known to appear in the fermionic case. Then, we can expand $\mathcal{L}$ in a series of the form

$$\mathcal{L} = \sum_{n=0}^{\infty} \lambda^{2n-2} \mathcal{L}_{2n}(T, \partial_\mu T) , \tag{3.3}$$

where

$$\mathcal{L}_{2n} = \sum_{l=0}^{n} a_l^{(n)} (\partial_\mu T \partial^\mu T)^l T^{2(n-l)} \tag{3.4}$$

and λ is a constant related to the normalization of the tachyon T. The problem of determining $\mathcal{L}$ reduces into the problem of determining the infinite set of constants $a_l^{(n)}$. This can be achieved by imposing on $\mathcal{L}$ the following two requirements that follow directly from the discussion in the introduction:

(A) The equations of motion of $\mathcal{L}$ admit the generic rolling tachyon profile (1.3) as an exact solution.

(B) The classical field theory $2n$-point amplitude reproduces exactly the Veneziano $2n$-point amplitudes (2.2) to leading (quadratic) order in spatial momenta, i.e. it reproduces (2.23) for any n.

The $2n$-th order equations of motion for (3.3) read [1]

$$\sum_{l=1}^{n} l a_l^{(n)} \partial^\mu \left[(\partial_\lambda T \partial^\lambda T)^{l-1} (\partial_\mu T) T^{2(n-l)} \right] = \sum_{l=0}^{n} (n-l) a_l^{(n)} T^{2(n-l)-1} (\partial_\mu T \partial^\mu T)^l \ . \tag{3.5}$$

By demanding that they admit

$$T = T_+ e^{\frac{i}{\sqrt{2}} x^0} + T_- e^{-\frac{i}{\sqrt{2}} x^0} \tag{3.6}$$

as an exact solution, we obtain at order $2n$ a set of n linear equations

$$a_l^{(n)} = \frac{(n-1)! 2^{l-1}}{(n-l)! l! (2l-1)} a_1^{(n)} \tag{3.7}$$

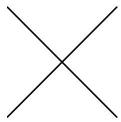
for the $n+1$ couplings $a_l^{(n)}$, $(l = 0, 1, ..., n)$. In this way, the first requirement reduces the problem of determining $\mathcal{L}$ drastically. At each order *only one* coefficient remains to be determined and this will be done by imposing condition (B). This requires an explicit calculation of the tree level field theory amplitudes to which we now turn. We begin with a few illustrating examples at 4th and 6th order.

3.1. Field theory 4-point amplitudes

At 4th order the field theory couplings are summarized by the following Lagrangian

$$\mathcal{L}_4 = \lambda^2 \big(a_0^{(2)} T^4 + a_1^{(2)} (\partial_\mu T \partial^\mu T) T^2 + a_2^{(2)} (\partial_\mu T \partial^\mu T)^2 \big). \tag{3.8}$$

Translating this expression into momentum space, we obtain the 4-point function (analogue of the string amplitude (2.13))

$$\mathcal{A}_4 =$$  $$= \langle T^{(+)}_{\vec{k}_1} T^{(+)}_{\vec{k}_2} T^{(-)}_{\vec{p}_1} T^{(-)}_{\vec{p}_2} \rangle =$$

$$\begin{aligned} &= \lambda^2 \big\{ 6a_0^{(2)} - a_1^{(2)} [(k_1 \cdot k_2) + (k_1 \cdot p_1) + (k_1 \cdot p_2) + (k_2 \cdot p_1) + (k_2 \cdot p_2) + (p_1 \cdot p_2)] + \\ &+ 2a_2^{(2)} [(k_1 \cdot k_2)(p_1 \cdot p_2) + (k_1 \cdot p_1)(k_2 \cdot p_2) + (k_1 \cdot p_2)(k_2 \cdot p_1)] \big\} \ . \end{aligned} \tag{3.9}$$

All momenta are on-shell and everything is considered in Euclidean signature (the inner product between two vectors reads $k \cdot p = k^0 p^0 + \vec{k} \cdot \vec{p}$). The expansion of this amplitude to quadratic order in spatial momenta gives

$$\mathcal{A}_4 = \lambda^2 \big(6a_0^{(2)} + a_1^{(2)} + \frac{3}{2} a_2^{(2)} \big) - \lambda^2 a_2^{(2)} \sum_{i,i'=1}^{2} u_{ii'} \ . \tag{3.10}$$

This expression has the same form as its string theory counterpart (2.17) and a direct comparison between them yields the following two conditions on the $\mathcal{L}_4$ coupling constants:

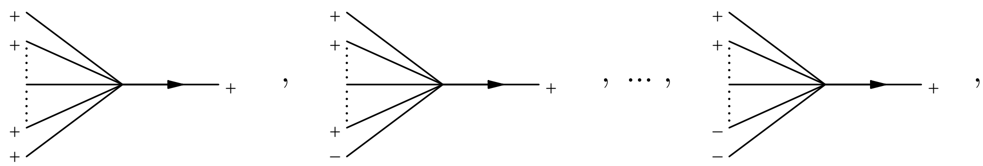
$$\begin{aligned} &6a_0^{(2)} + a_1^{(2)} + \frac{3}{2} a_2^{(2)} = 0, \\ &a_2^{(2)} = \lambda^{-2} g_o^2 \frac{\pi^2}{4} \ . \end{aligned} \tag{3.11}$$

The first condition comes from the vanishing of the constant term in the string theory amplitude (2.13), which follows from the fact that the rolling tachyon profile is exactly marginal. In field theory, the analogous requirement (condition (A) above) gives at the $2n$-th order n constraints, which lead to the recursion relation (3.7). For $n = 2$ one of these constraints should be coincident with the first equation in (3.11). We can see this, generalized to any order, in the following way.

Substitute the rolling tachyon profile (3.6) into the equations of motion (3.5) and set the coefficients of the n exponentials $e^{i \frac{m}{\sqrt{2}} x^0}$ (for $m = 1, 3, ..., 2n-1$) to zero.[6] These

[6] There is an extra set of equations originating from the vanishing of the coefficients of the exponentials $e^{-i \frac{m}{\sqrt{2}} x^0}$ $(m = 1, 3, ..., 2n-1)$. This set is identical to that of the positive frequency exponentials and will not be discussed independently.

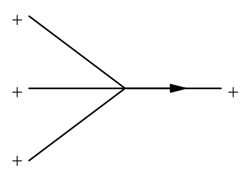
equations effectively set to zero all tree-level $2n$-point vertices of the form



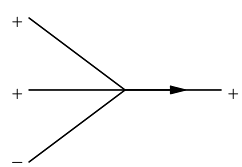
(3.12)

with the last rightmost vertex having n positive incoming frequencies and $n-1$ negative incoming frequencies. All these vertices (in total n in number) are evaluated at zero spatial momenta and they provide a set of equations equivalent to the recursion relations (3.7). The momentum of the outgoing particle is determined by momentum conservation and can be on-shell only for the last rightmost diagram, where the positive incoming frequencies almost cancel the negative incoming frequencies. This latter condition is the only one appearing in the evaluation of on-shell amplitudes.

To illustrate this point more explicitly let us consider in detail the example of the 4-point vertices. For $n = 2$ we find two vertices, which upon evaluation give



$$= \lambda^2 \big(2a_0^{(2)} + a_1^{(2)} - \frac{3}{2} a_2^{(2)} \big), \tag{3.13}$$



$$= \lambda^2 \big(6a_0^{(2)} + a_1^{(2)} + \frac{3}{2} a_2^{(2)} \big). \tag{3.14}$$

The second condition is the only one appearing in the on-shell 4-point amplitude (3.10). The vanishing of the first vertex (3.13) is an off-shell statement, which will be imposed here as part of condition (A) of the previous subsection. We see that conditions (A) and (B) are not completely independent, but they have a small overlap.

At non-zero spatial momenta comparison with the on-shell string theory amplitude provides one more equation, the second one in (3.11). In higher orders this extra equation, together with the n ones coming from condition (A), is enough to determine the exact form of the effective Lagrangian order by order. In 4th order, we simply get a relation between the coefficient $a_2^{(2)}$, the normalization factor of the tachyon λ^2 and the open string coupling g_o.

3.2. Field theory 6-point amplitudes

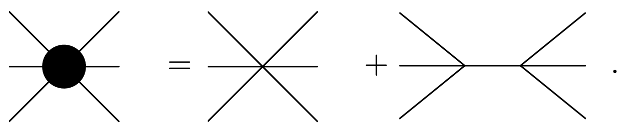
The field theory 6-point amplitude

$$\mathcal{A}_6 = \langle T^{(+)}_{\vec{k}_1} T^{(+)}_{\vec{k}_2} T^{(+)}_{\vec{k}_3} T^{(-)}_{\vec{p}_1} T^{(-)}_{\vec{p}_2} T^{(-)}_{\vec{p}_3} \rangle \tag{3.15}$$

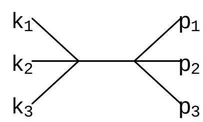
receives several contributions. One comes directly from the 6th order part of the effective tachyon Lagrangian

$$\mathcal{L}_6 = \lambda^4 \sum_{l=0}^{3} a_l^{(3)} (\partial_\mu T \partial^\mu T)^l T^{6-2l} \tag{3.16}$$

and the rest come from exchange diagrams involving 4th order vertices. In diagrammatic form we have

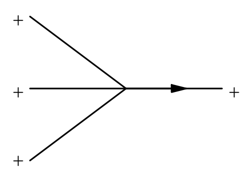
$\mathcal{A}_6 =$  (3.17)

There are two types of exchange diagrams. The first one



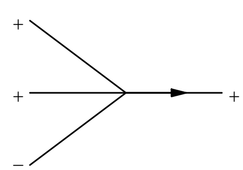
(3.18)

involves 4-vertices of the form



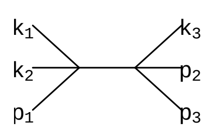
(3.19)

and the propagator is always off-shell and regular even when the spatial momenta vanish. The second type involves 4-vertices of the form



, (3.20)

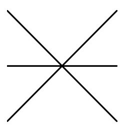
which become on-shell as we send the spatial momenta to zero and develop a pole. A particular example includes the diagram



. (3.21)

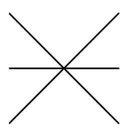
There are nine different choices of this type and each of them contributes non-polynomial terms in 6th order, which scale like $(\text{momentum})^4/(\text{momentum})^2$ to leading order in spatial momenta. Analogous exchange diagrams appeared in string theory (see eq. (2.21)). A more detailed analysis provides the following information.

The 6th order vertex arising from (3.16) (here we demonstrate explicitly only a few representative terms) reads

λ^{-4} 

$$
\begin{aligned}
&= \frac{1}{20}\Big[6!a_0^{(3)} - 2\cdot 4!a_1^{(3)}\big(k_1\cdot(k_2+k_3+p_1+p_2+p_3)+ \\
&\quad + k_2\cdot(k_3+p_1+p_2+p_3) + k_3\cdot(p_1+p_2+p_3) + p_1\cdot(p_2+p_3) + p_2\cdot p_3\big)+ \\
&\quad + 16a_2^{(3)}\big((k_1\cdot k_2)(k_3\cdot(p_1+p_2+p_3)) + \cdots\big)- \\
&\quad - a_3^{(3)}\big((k_1\cdot k_2)(k_3\cdot p_1)(p_2\cdot p_3) + \cdots\big)\Big] \ .
\end{aligned}
\tag{3.22}
$$

Simple symmetry considerations (similar to those presented for string theory amplitudes above) reduce the form of this expression in the limit of small spatial momenta to

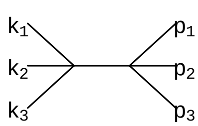


$$
= t_6 + s_6 \sum_{i,i'} u_{ii'} \ . \tag{3.23}
$$

t_6 and s_6 are certain constants that can be determined by a straightforward calculation

$$
\begin{aligned}
t_6 &= \frac{\lambda^4}{20}\big(6!a_0^{(3)} + 3\cdot 4!a_1^{(3)} + 36a_2^{(3)} + \frac{6!}{8}a_3^{(3)}\big), \\
s_6 &= \lambda^4\big(-2a_2^{(3)} + 9a_3^{(3)}\big) \ .
\end{aligned}
\tag{3.24}
$$

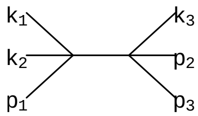
The regular exchange diagrams can be expanded to quadratic order in a similar fashion. We find:



$$
\begin{aligned}
&= \frac{9}{4}\lambda^4\big(2a_0^{(2)} + a_1^{(2)} - \frac{3}{2}a_2^{(2)}\big)^2 + \\
&+ \frac{3}{64}\lambda^4\big(2a_0^{(2)} + a_1^{(2)} - \frac{3}{2}a_2^{(2)}\big)\big(12a_0^{(2)} - 2a_1^{(2)} + 31a_2^{(2)}\big)\sum_{i,i'} u_{ii'} \ .
\end{aligned}
\tag{3.25}
$$

Both terms in this expression are proportional to the linear combination $2a_0^{(2)} + a_1^{(2)} - \frac{3}{2}a_2^{(2)}$ appearing in (3.13). This combination is set to zero by requirement (A) and the regular set of exchange diagrams does not contribute in 6th order.

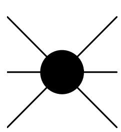
The second type of exchange diagrams involves a nearly on-shell pole. Let us consider explicitly one of them, say the one appearing in (3.21). To leading order in spatial momenta we find:



$$= \frac{1}{-s_{12}+u_{11'}+u_{22'}}\Big[\big(6a_0^{(2)} + a_1^{(2)} + \frac{3}{2}a_2^{(2)}\big)^2 + \\ + (-s_{12}+u_{11'}+u_{22'})\big(6a_0^{(2)} + a_1^{(2)} + \frac{3}{2}a_2^{(2)}\big)\big(a_1^{(2)} + 3a_2^{(2)}\big) \\ + (-s_{12}+u_{11'}+u_{22'})^2\big(\frac{1}{4}\big(a_1^{(2)}\big)^2 + a_1^{(2)}a_2^{(2)} + \frac{5}{2}\big(a_2^{(2)}\big)^2\big) + \\ + (-s_{12}+u_{11'}+u_{22'})(s_{12}+t_{23})\big(\big(a_2^{(2)}\big)^2 - a_1^{(2)}a_2^{(2)}\big) + \\ + 4\big(a_2^{(2)}\big)^2 s_{12}t_{23}\Big] \ . \tag{3.26}$$

The first two terms are proportional to the linear combination $6a_0^{(2)} + a_1^{(2)} + \frac{3}{2}a_2^{(2)}$, which has been set to zero (see eq. (3.11)). The third and fourth terms are non-vanishing polynomial quadratic contributions and the last one is a pole contribution that scales like (momentum)^4/(momentum)^2. Using eqs. (3.11), (3.26) and (2.22), we can check that the same non-polynomial term appears in string theory as part of the function W_6.

The full 6th order amplitude is obtained by summing over the individual contributions (3.23), (3.26):



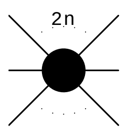
$$= \lambda^4\Big[\frac{1}{20}\big(6!a_0^{(3)} + 3\cdot 4!a_1^{(3)} + 36a_2^{(3)} + \frac{6!}{8}a_3^{(3)}\big) + \\ + \Big(-2a_2^{(3)} + 9a_3^{(3)} + \frac{1}{8}\big(a_1^{(2)}\big)^2 - \frac{13}{4}a_1^{(2)}a_2^{(2)} + \frac{25}{8}\big(a_2^{(2)}\big)^2\Big)\sum_{i,i'} u_{ii'} + \\ + 4\big(a_2^{(2)}\big)^2\Big(\frac{s_{12}t_{23}}{-s_{12}+u_{11'}+u_{22'}} + \cdots\Big)\Big] \ . \tag{3.27}$$

In the derivation of this expression we have enforced the 4th order vanishing conditions (3.13), (3.14) and by $\cdots$ in the last term a summation over the remaining eight channels of the type (3.21) is implied. We can further check the vanishing of the constant term $6!a_0^{(3)} + 3\cdot 4!a_1^{(3)} + 36a_2^{(3)} + \frac{6!}{8}a_3^{(3)}$ as a consequence of the recursion relation (3.7).

In conclusion, we see explicitly that the structure of the 6th order tree-level amplitudes in field theory is precisely the same to the one encountered in string theory above. The leading term in spatial momenta scales like (momentum)^2 and breaks up into a quadratic polynomial piece and a rational function (denoted by W_6 in (2.20)) coming from exchange diagrams.

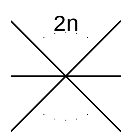
3.3. Field theory 2n-point amplitudes

The discussion of the 6th order amplitudes can be extended naturally to any order. In the limit of vanishing spatial momenta the full $2n$-point amplitudes in field theory take the form



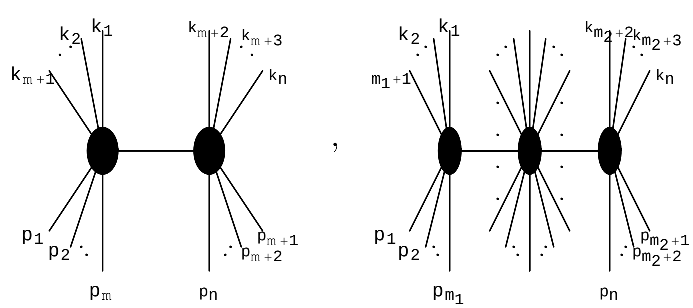
$$= \tilde{C}_{2n} \sum_{i,i'} u_{ii'} + \tilde{W}_{2n}(\vec{k}_i, \vec{p}_j) \ . \tag{3.28}$$

A possible constant term vanishes after imposing the conditions coming from requirement (A) (see eq. (3.7)) up to order $2n$. $\tilde{C}_{2n}$ is a constant arising from the leading order expansion of two sets of diagrams. The first (regular) set includes the irreducible $2n$-vertex



(3.29)

and exchange diagrams, which are analogs of (3.18). The propagators appearing in these diagrams are regular in the special kinematical regime of this paper and yield a finite contribution. Their leading order momentum dependence is a quadratic polynomial. The second set includes a series of exchange diagrams with $k \neq 0$ near on-shell propagators. Particular examples include the diagrams



(3.30)

with $k = 1, 2$ respectively. These diagrams are higher order analogs of those appearing in (3.21). The leading order momentum dependence of such diagrams can be summarized by a quadratic polynomial, which contributes to the constant $\tilde{C}_{2n}$, and a rational function of the external momenta, which contributes to the function $\tilde{W}_{2n}$ in the same way as in string theory. The poles appearing in this function are identical to the poles appearing in the string theory function W_{2n} and the residues are derived in both cases as appropriate factorizations of the same lower order amplitudes. This implies the relation

$$\tilde{W}_{2n} = W_{2n} \tag{3.31}$$

for any n. We verified this relation explicitly for $n = 2, 3$ above.

3.4. Computation of the effective action

We have demonstrated that in field theory the leading order amplitudes take the form

$$\mathcal{A}_{2n} = \left(\sum_{l=1}^{n} d_l^{(n)} a_l^{(n)} + q_{\text{lower order}}^{(n)} \right) \sum_{i,i'} u_{ii'} + W_{2n}(\vec{k}_i, \vec{p}_j) \ , \tag{3.32}$$

with $d_l^{(n)}$ certain constants that arise from the field theory vertex (3.29) (see appendix A) and $q_{\text{lower order}}^{(n)}$ a constant that arises from lower order exchange diagrams. In string theory the respective expression reads

$$\mathcal{A}_{2n} = C_{2n} \sum_{i,i'} u_{ii'} + W_{2n}(\vec{k}_i, \vec{p}_j) \ , \tag{3.33}$$

for some constant C_{2n}. The same function W_{2n} appears on both sides and the task of matching the two results amounts to matching the coefficients of the $\sum_{i,i'} u_{ii'}$ term. This gives an extra condition at the $2n$-th order in field theory, which fixes the full set of $n+1$ couplings $a_l^{(n)}$ in (3.4) and determines the form of the effective field theory action.

Indeed, the other n conditions among the couplings $a_l^{(n)}$ arise from condition (A) in the beginning of this section. They are summarized in relation (3.7). One could use this relation to re-express the sum $\sum_{l=1}^{n} d_l^{(n)} a_l^{(n)}$ appearing in (3.32) in terms of just one coefficient, say $a_1^{(n)}$ and the resulting expression can be written as $d^{(n)} a_1^{(n)}$ for an appropriate constant $d^{(n)}$. Then, matching equations (3.32) and (3.33) amounts to setting

$$d^{(n)} \ a_1^{(n)} + q_{\text{lower order}}^{(n)} = C_{2n} \ . \tag{3.34}$$

This determines the unknown coupling constant $a_1^{(n)}$ in terms of $d^{(n)}, q^{(n)}_{\text{lower order}}$ and C_{2n}. Subtleties could arise if, for some n, we happened to find $d^{(n)} = 0$. In that case eq. (3.34) would not be satisfied for any value of $a_1^{(n)}$, unless $q^{(n)}_{\text{lower order}} = C_{2n}$. An explicit alternating series expression for $d^{(n)}$ is given in appendix A. Using Mathematica, we have checked that up to $n = 15$ the absolute value of $d^{(n)}$ is a non-vanishing monotonically increasing function of n. We expect a similar statement to be true for all n.

In summary, the above discussion demonstrates the existence of a tachyon effective action with a prescribed set of properties. The form of this action can be fixed uniquely (modulo the usual field redefinition ambiguities) so long as we know the precise value of the constants $d^{(n)}, q^{(n)}_{\text{lower order}}$ and C_{2n}. This turns out to be a non-trivial technical problem involving, in particular, the computation of a series of complicated integrals (see, for example, (2.19)).

4. Effective actions in the presence of tachyon condensates

The analysis of the previous sections established that up to field redefinitions there exists a unique tachyon effective action satisfying requirements (A) and (B) of section 3 in the absence of a tachyon condensate. Can we trust the same action also in the presence of a non-zero tachyon condensate? For starters, let us consider turning on a non-zero constant T_+ in the rolling tachyon profile (3.6), while keeping T_- zero. In the presence of this half-brane rolling tachyon the spectrum of scaling dimensions of the theory remains unmodified. If the theory can be treated perturbatively in T_+, the effective action of section 3 can still be used to describe the leading terms of the string theory scattering amplitudes (1.9) in the limit of small spatial momenta and this extends its validity in the presence of a non-vanishing T_+-condensate.

On a practical level, it would be useful to have more than a statement about the existence of this action. It would be useful to obtain the exact form of the Lagrangian (3.3), (3.4). This is possible in the language of section 3, but it involves a series of complicated calculations. In order to avoid these technical difficulties, the analysis of [1] proposed that the exact form of the effective action (3.3), (3.4) can be fixed with an alternative calculation. This approach is based on the evaluation of the "unintegrated" disc partition function in Minkowskian signature. Let us briefly explain the basic features of this computation. On general grounds, one expects that the on-shell spacetime action is equal to the perturbed

disc partition sum [5,14-19]. By stripping off the integral over the zero-mode x^0, it is natural to conjecture the closely related formula

$$\mathcal{L}_{\text{on-shell}}(x^0) = Z'(x^0) \ . \tag{4.1}$$

$\mathcal{L}_{\text{on-shell}}(x^0)$ is the on-shell value of the effective Lagrangian and $Z'(x^0)$ is the disc partition sum

$$Z'(x^0) = \int [dx'^0] \ e^{-\mathcal{S}_{\text{bulk}} - \int_{\partial D} d\sigma T(x^0)} \tag{4.2}$$

where the disc path integral is performed only over the non-zero modes x'^0; the zero modes remain unintegrated and momentum conservation in the x^0 direction is not imposed. In our case, both sides of (4.1) should be evaluated on the half-brane tachyon profile $T = T_+ e^{\frac{1}{\sqrt{2}}x^0}$. The value of $Z'(x^0)$ for this profile has been computed in [20]:

$$Z'(x^0) = \frac{1}{1 + \frac{1}{2}T_+^2 e^{\sqrt{2}x^0}} \ . \tag{4.3}$$

With this information one can show, by a direct application of (4.1), that the effective Lagrangian takes the form [1]

$$\mathcal{L} = -\frac{1}{1+\frac{1}{2}T^2}\sqrt{1+\frac{1}{2}T^2 + \partial_\mu T \partial^\mu T} \ , \tag{4.4}$$

which transforms into the more familiar tachyon DBI action[7] [21-46]

$$\mathcal{L} = -\frac{1}{\cosh \frac{\tilde{T}}{\sqrt{2}}}\sqrt{1 + \partial_\mu \tilde{T} \partial^\mu \tilde{T}} \tag{4.5}$$

after the field redefinition

$$\frac{T}{\sqrt{2}} = \sinh \frac{\tilde{T}}{\sqrt{2}} \ . \tag{4.6}$$

This approach is similar to the usual derivation of the massless Born-Infeld action. In that case, one is instructed to compute the disc partition sum in the presence of a constant $F_{\mu\nu}$ profile. It is natural to expect that this latter derivation of the tachyon effective action is equivalent to the one described in section 3 and that both of them lead to the

[7] As noted in [1] the form of this Lagrangian is quite different from the one found in boundary string field theory (BSFT) [15,16,17]. From the point of the present discussion this is quite natural. The BSFT action is valid for deeply off-shell tachyons, e.g. of the form $T = a + ux^2$ in the bosonic case, while the tachyon DBI action is valid for nearly on-shell configurations of the form (1.3).

same result. It would be interesting to compute the constants $C_{2n}, \tilde{C}_{2n}$ in (2.23) and (3.28) respectively and check this claim explicitly. This would also provide a non-trivial check of the assumption (4.1).

More generally, we can ask whether the same effective action is valid in the presence of other non-vanishing tachyon condensates. For example, consider turning on both T_+ and T_- in (1.3). In that case several complications appear. One drastic effect of the condensate is the modification of the spectrum. Another related effect, most apparent in the Minkowskian signature, is the absence of asymptotic flat regions in spacetime. At very early ($x^0 \to -\infty$) and very late ($x^0 \to +\infty$) times the system is very far from its perturbative open string vacuum and the meaning of S-matrix elements is far from obvious. In fact, as $|T| \to \infty$ the theory approaches the closed string vacuum, where no physical open string excitations are believed to survive. These effects invalidate the action of section 3 as a reliable tachyon effective action. Indeed, this breakdown has been observed directly in the computation of the stress-energy tensor in refs. [27,38,1]. Despite of these problems, the action (4.5) continues to be valid at late times. As $x^0 \to +\infty$ the exponential term $T_- e^{-\frac{1}{\sqrt{2}}x^0}$ can be ignored in the rolling tachyon profile (1.3) and the action (4.5) resurfaces at late times as a valid description of the appropriate string theory dynamics.

It would be very interesting to see whether it is possible to obtain another effective action in the presence of the full-brane tachyon condensate. If such an action exists it will not be Poincare (Euclidean) invariant. Poincare invariance is explicitly broken by the full-brane condensate.[8] The band structure of the open string theory spectrum in the presence of the full-brane tachyon [10] suggests that the desired action could be some kind of non-commutative deformation - possibly a q-deformation - of the original tachyon DBI action (4.5). Whether this expectation is true remains to be seen.

5. Conclusions and open problems

What are the consequences of the present construction for open string tachyon condensation? Building on previous work [1], we argued that the dynamics of the open string tachyon in the vicinity of the homogeneous half-brane solution are captured correctly by the effective action (4.5). The validity of this effective action is not restricted only at

[8] The half-brane condensate allows for a manifestly Poincare invariant flat region in the far past and does not break Poincare invariance.

late times, where the gradients of the tachyon potential vanish asymptotically. The action describes correctly the process of tachyon condensation everywhere along the decay. By that we mean precisely the following. As long as the initial conditions are such that the tachyon never deviates considerably away from the homogeneous half-brane profile (i.e. the tachyon has the general inhomogeneous form $T = T_+(x^\mu)e^{\frac{1}{\sqrt{2}}x^0} + T_-(x^\mu)e^{-\frac{1}{\sqrt{2}}x^0}$, with $|\partial_\mu T(x^\nu)| \ll 1, |T_-(x^\nu)| \ll 1$) the effective action (4.5) provides a valid description of the dynamics and will not break down. This is also in nice agreement with some features of general inhomogeneous solutions of this action discussed in refs. [47,48] (for more details see [1]).

The usefulness of the action (4.5) is comparable to that of the usual massless DBI action. Both actions capture reliably a class of phenomena in the full open string theory that can be associated to small deviations away from the corresponding exactly marginal profiles. It is worth mentioning the following two prominent examples in the tachyon case (again, for more details we refer the reader to [1]):

(*i*) The tachyon DBI action (4.5) reproduces the correct stress-energy tensor in homogeneous tachyon decay for the half-brane solution and the leading T_- behaviour of the stress-energy tensor for the full-brane solution.

(*ii*) It contains solitonic solutions corresponding to lower dimensional stable D-branes [49]. Small excitations of these solitons are massless fields (similar to the gauge fields A_μ and scalars Y^I on a stable D-brane) and one can check explicitly [49] that the effective action describing the dynamics of these modes is the massless DBI action, exactly as anticipated from string theory.

We would like to conclude with a few interesting open problems. One of them is the extension of the action (4.5) to include the gauge field A_μ and the massless scalars Y^I parameterizing the position of the D-brane. This is particularly simple when $F_{\mu 0} = \partial_0 Y^I = 0$. In that case the rolling tachyon solution (1.3) is not modified by the expectation values of the massless fields and the full action is very likely to be given by the DBI generalization

$$\mathcal{L} = -\frac{1}{\cosh\frac{\tilde{T}}{2}}\sqrt{-\mathrm{det}G} \ , \tag{5.1}$$

with

$$G_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \tilde{T}\partial_\nu \tilde{T} + \partial_\mu Y^I \partial_\nu Y^I + F_{\mu\nu} \ . \tag{5.2}$$

In the presence of a nonvanishing electric field $F_{\mu 0}$ and non-vanishing D-brane velocities $\partial_0 Y^I$ the above extension is less straightforward. Presumably one has to replace (1.3) by

a solution of the tachyon equations of motion in the open string metric and proceed from there. For a recent discussion in favor of the action (5.1), (5.2) see [50].

The case of a non-zero electric field is also interesting for another reason.[9] A constant electric field couples on the boundary of the disc with the coupling

$$\int_{\partial\Sigma} d\sigma\ F_{i0} x^i \partial x^0 \ . \tag{5.3}$$

The Euclidean version of the x^0 CFT has an algebraic $SU(2)$ symmetry under which $i\partial x^0$ rotates, for example, into $e^{\frac{i}{\sqrt{2}}x^0} + e^{-\frac{i}{\sqrt{2}}x^0}$. This suggests that (5.3) has the following equivalent form[10]

$$\int_{\partial\Sigma} d\sigma\ F_{i0} x^i \left(e^{\frac{i}{\sqrt{2}}x^0} + e^{-\frac{i}{\sqrt{2}}x^0}\right) \ . \tag{5.4}$$

This is a (linear) inhomogeneous version of the full-brane rolling tachyon solution. A similar linear inhomogeneous version of the half-brane rolling tachyon solution has been considered in a recent paper [51]. It would be interesting to explore the properties of open string theory in the presence of such condensates and see to what extent the algebraic $SU(2)$ symmetry is a useful tool.

Finally, it would be very interesting to see if the general point of view on effective actions reviewed here is useful for thinking about analogous examples in closed string tachyon condensation. Progress in this direction requires establishing the existence of an exact closed string tachyon solution analogous to the rolling tachyon solution in open string theory. For a previous suggestion that the perturbation

$$\delta\mathcal{L}_{\rm ws} = \lambda e^{2x^0} \tag{5.5}$$

is exactly marginal see ref. [52].

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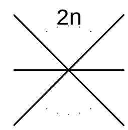
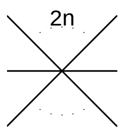
[9] I would like to thank D. Kutasov for suggesting this interesting point.

[10] The presence of the x^i CFT should not affect this statement. We could apply the $SU(2)$ symmetry of the x^0 CFT perturbatively in F_{0i}.

Appendix A. Calculation of the field theory amplitude 

In this appendix we compute the coefficient of the $\sum_{i,i'} u_{ii'}$ term in the field theory amplitude 

at order $2n$. This computation can be achieved compactly in the following way. Set

$$T = \sum_{i=1}^{n} (e^{ik_i \cdot x} + e^{ip_i \cdot x}) \tag{A.1}$$

into the field theory Lagrangian

$$\mathcal{L}_{2n} = \sum_{l=0}^{n} a_l^{(n)} \left(\partial_\mu T \partial^\mu T\right)^l T^{2(n-l)} \tag{A.2}$$

and pick up those terms that satisfy the momentum conservation constraints $\sum_i (k_i + p_i) = 0$. For simplicity, we have omitted a symmetry factor and the normalization constant λ^{2n-2}. The resulting expression can be expanded to quadratic order and, as we know on general grounds, the final result will take the form

$$t_{2n} + s_{2n} \sum_{ii'} u_{ii'} \ . \tag{A.3}$$

We would like to determine the coefficient s_{2n}. A convenient choice of the external momenta simplifies the computation considerably. If we set

$$\begin{aligned} k_1 &= (\frac{1}{\sqrt{2}}(1-\sigma^2), \sigma, 0, ...), \ k_2 = k_3 = ... = k_n = (\frac{1}{\sqrt{2}}, 0, 0, ...), \\ p_1 &= (-\frac{1}{\sqrt{2}}(1-\sigma^2), -\sigma, 0, ...), \ p_2 = p_3 = ... = p_n = (-\frac{1}{\sqrt{2}}, 0, 0, ...) \end{aligned} \tag{A.4}$$

and substitute (A.1) into (A.2), we find

$$\begin{aligned} \mathcal{L}_{2n} = \sum_{l=0}^{n} a_l^{(n)} \Bigg[& \frac{1}{2} \big(e^{ik_1 \cdot x} - e^{-ik_1 \cdot x} + (n-1)\big(e^{ik_2 \cdot x} - e^{-ik_2 \cdot x}\big)\big)^2 - \\ & - (n-1)\sigma^2 \big(e^{ik_1 \cdot x} - e^{-ik_1 \cdot x}\big)\big(e^{ik_2 \cdot x} - e^{-ik_2 \cdot x}\big) \Bigg]^l \times \\ & \times \left[e^{ik_1 \cdot x} + e^{-ik_1 \cdot x} + (n-1)\big(e^{ik_2 \cdot x} + e^{-ik_2 \cdot x}\big) \right]^{2(n-l)} . \end{aligned} \tag{A.5}$$

We expand to linear order in σ^2 and drop the σ-independent terms to obtain

$$\begin{aligned}\mathcal{L}_{2n} \sim \sigma^2 \sum_{l=1}^{n} a_l^{(n)} (n-1) \frac{l}{2^{l-1}} \Big[- e^{i(k_1+k_2)\cdot x} - e^{-i(k_1+k_2)\cdot x} + e^{i(k_1-k_2)\cdot x} + e^{-i(k_1-k_2)\cdot x} \Big] \times \\ \times \Big[e^{ik_1\cdot x} - e^{-ik_1\cdot x} + (n-1)\big(e^{ik_2\cdot x} - e^{-ik_2\cdot x}\big) \Big]^{2l-2} \times \\ \times \Big[e^{ik_1\cdot x} + e^{-ik_1\cdot x} + (n-1)\big(e^{ik_2\cdot x} + e^{-ik_2\cdot x}\big) \Big]^{2(n-l)} .\end{aligned} \tag{A.6}$$

Repeated use of the binomial expansion

$$(a+b)^m = \sum_{r=0}^{m} \binom{m}{r} a^{m-r} b^r \tag{A.7}$$

and the identity

$$\sum_{i,i'} u_{ii'} = 2(n-1)\sigma^2 \tag{A.8}$$

provides the desired coefficient

$$\begin{aligned}s_{2n} = \frac{1}{2}\sigma^2 \sum_{l=1}^{n} \sum_{r_1=0}^{2(n-l)} \sum_{r_2=0}^{2(n-l)-r_1} \sum_{r_3=0}^{r_1} \sum_{s_1=0}^{2l-2} \sum_{s_2=0}^{2l-2-s_1} \sum_{s_3=0}^{s_1} a_l^{(n)} \frac{l}{2^{l-1}} (-)^{n-r_2-r_3-1} (n-1)^{r_1+s_1} \times \\ \times \binom{2(n-l)}{r_1} \binom{2(n-l)-r_1}{r_2} \binom{r_1}{r_3} \binom{2l-2}{s_1} \binom{2l-2-s_1}{s_2} \binom{s_1}{s_3} \times \\ \times \Big[\delta_{s_2, n-r_2-\frac{1+r_1+s_1}{2}} \delta_{s_3, \frac{1+r_1+s_1}{2}-r_3} + \delta_{s_2, n-r_2-\frac{3+r_1+s_1}{2}} \delta_{s_3, \frac{-1+r_1+s_1}{2}-r_3} + \\ + \delta_{s_2, n-r_2-\frac{1+r_1+s_1}{2}} \delta_{s_3, \frac{-1+r_1+s_1}{2}-r_3} + \delta_{s_2, n-r_2-\frac{3+r_1+s_1}{2}} \delta_{s_3, \frac{1+r_1+s_1}{2}-r_3} \Big] ,\end{aligned} \tag{A.9}$$

with the Kronecker δ terms in the last parenthesis enforcing the momentum conservation conditions. For $n = 2, 3$ we reproduce the results of sections 3.1 and 3.2 respectively.[11] This provides a trivial check of (A.9). Moreover, we can show by direct computation of (A.9), or by simple inspection of (A.5) that the coefficient of $a_1^{(n)}$, denoted by $d_1^{(n)}$ in (3.32), vanishes identically for any n. The remaining coefficients $d_l^{(n)}$ are generically non-zero.

[11] In this comparison the appropriate symmetry factors should be reinstated (1/4 for $n = 2$ and 1/20 for $n = 3$).

For the computation of the tachyon effective action in section 3.4 it was important to establish the non-vanishing of the overall coefficient s_{2n}, or $d^{(n)}$ in (3.34). Substituting the recursion relation (3.7) into (A.9) we find $d^{(n)}$ in terms of the alternating series

$$d^{(n)} = \frac{1}{2}\sum_{l=1}^{n}\sum_{r_1=0}^{2(n-l)}\sum_{r_2=0}^{2(n-l)-r_1}\sum_{r_3=0}^{r_1}\sum_{s_1=0}^{2l-2}\sum_{s_2=0}^{2l-2-s_1}\sum_{s_3=0}^{s_1}(-)^{n-1-r_2-r_3}(n-1)^{r_1+s_1}\times$$
$$\times\frac{(n-1)!(2(n-l))!(2l-2)!}{(n-l)!(l-1)!(2l-1)}\frac{1}{r_2!(2(n-l)-r_1-r_2)!}\frac{1}{r_3!(r_1-r_3)!}\frac{1}{s_2!(2l-2-s_1-s_2)!}\times$$
$$\times\frac{1}{s_3!(s_1-s_3)!}\left[\delta_{s_2,n-r_2-\frac{1+r_1+s_1}{2}}\delta_{s_3,\frac{1+r_1+s_1}{2}-r_3}+\delta_{s_2,n-r_2-\frac{3+r_1+s_1}{2}}\delta_{s_3,\frac{-1+r_1+s_1}{2}-r_3}+\right.$$
$$\left.+\,\delta_{s_2,n-r_2-\frac{1+r_1+s_1}{2}}\delta_{s_3,\frac{-1+r_1+s_1}{2}-r_3}+\delta_{s_2,n-r_2-\frac{3+r_1+s_1}{2}}\delta_{s_3,\frac{1+r_1+s_1}{2}-r_3}\right]. \tag{A.10}$$

We have not been able to find a considerable simplification of this formula that would demonstrate immediately the non-vanishing of $d^{(n)}$ for all n. Instead, we have checked explicitly, using Mathematica, that up to $n = 15$ $\left|d^{(n)}\right|$ is a non-vanishing monotonically increasing function of n. We expect that a similar statement holds for all n.

References

[1] D. Kutasov and V. Niarchos, "Tachyon effective actions in open string theory," Nucl. Phys. B **666**, 56 (2003) [arXiv:hep-th/0304045].

[2] A. Sen, "Remarks on tachyon driven cosmology," arXiv:hep-th/0312153.

[3] A. A. Tseytlin, "Born-Infeld action, supersymmetry and string theory," arXiv:hep-th/9908105.

[4] A. A. Tseytlin, "Vector Field Effective Action In The Open Superstring Theory," Nucl. Phys. B **276**, 391 (1986) [Erratum-ibid. B **291**, 876 (1987)].

[5] A. A. Tseytlin, "Sigma Model Approach To String Theory," Int. J. Mod. Phys. A **4**, 1257 (1989).

[6] A. A. Tseytlin, "Sigma model approach to string theory effective actions with tachyons," J. Math. Phys. **42**, 2854 (2001) [arXiv:hep-th/0011033].

[7] A. Sen, "Rolling tachyon," JHEP **0204**, 048 (2002) [arXiv:hep-th/0203211].

[8] M. Smedback, "On effective actions for the bosonic tachyon," JHEP **0311**, 067 (2003) [arXiv:hep-th/0310138].

[9] C. G. Callan and I. R. Klebanov, "Exact C = 1 boundary conformal field theories," Phys. Rev. Lett. **72**, 1968 (1994) [arXiv:hep-th/9311092].

[10] C. G. Callan, I. R. Klebanov, A. W. Ludwig and J. M. Maldacena, "Exact solution of a boundary conformal field theory," Nucl. Phys. B **422**, 417 (1994) [arXiv:hep-th/9402113].

[11] J. Polchinski and L. Thorlacius, "Free Fermion Representation Of A Boundary Conformal Field Theory," Phys. Rev. D **50**, 622 (1994) [arXiv:hep-th/9404008].

[12] J. Polchinski, "String Theory. Vol. 2: Superstring Theory And Beyond," Cambridge, UK: Univ. Pr. (1998) 531 p.

[13] P. Di Francesco and D. Kutasov, "World sheet and space-time physics in two-dimensional (Super)string theory," Nucl. Phys. B **375**, 119 (1992) [arXiv:hep-th/9109005].

[14] E. Witten, "On background independent open string field theory," Phys. Rev. D **46**, 5467 (1992) [arXiv:hep-th/9208027].

[15] A. A. Gerasimov and S. L. Shatashvili, "On exact tachyon potential in open string field theory," JHEP **0010**, 034 (2000) [arXiv:hep-th/0009103].

[16] D. Kutasov, M. Marino and G. W. Moore, "Some exact results on tachyon condensation in string field theory," JHEP **0010**, 045 (2000) [arXiv:hep-th/0009148].

[17] D. Kutasov, M. Marino and G. W. Moore, "Remarks on tachyon condensation in superstring field theory," arXiv:hep-th/0010108.

[18] M. Marino, "On the BV formulation of boundary superstring field theory," JHEP **0106**, 059 (2001) [arXiv:hep-th/0103089].

[19] V. Niarchos and N. Prezas, "Boundary superstring field theory," Nucl. Phys. B **619**, 51 (2001) [arXiv:hep-th/0103102].

[20] F. Larsen, A. Naqvi and S. Terashima, "Rolling tachyons and decaying branes," JHEP **0302**, 039 (2003) [arXiv:hep-th/0212248].

[21] A. Sen, "Supersymmetric world-volume action for non-BPS D-branes," JHEP **9910**, 008 (1999) [arXiv:hep-th/9909062].

[22] M. R. Garousi, "Tachyon couplings on non-BPS D-branes and Dirac-Born-Infeld action," Nucl. Phys. B **584**, 284 (2000) [arXiv:hep-th/0003122].

[23] E. A. Bergshoeff, M. de Roo, T. C. de Wit, E. Eyras and S. Panda, "T-duality and actions for non-BPS D-branes," JHEP **0005**, 009 (2000) [arXiv:hep-th/0003221].

[24] J. Kluson, "Proposal for non-BPS D-brane action," Phys. Rev. D **62**, 126003 (2000) [arXiv:hep-th/0004106].

[25] G. W. Gibbons, K. Hori and P. Yi, "String fluid from unstable D-branes," Nucl. Phys. B **596**, 136 (2001) [arXiv:hep-th/0009061].

[26] N. D. Lambert and I. Sachs, "On higher derivative terms in tachyon effective actions," JHEP **0106**, 060 (2001) [arXiv:hep-th/0104218].

[27] A. Sen, "Tachyon matter," JHEP **0207**, 065 (2002) [arXiv:hep-th/0203265].

[28] A. Sen, "Field theory of tachyon matter," Mod. Phys. Lett. A **17**, 1797 (2002) [arXiv:hep-th/0204143].

[29] S. Sugimoto and S. Terashima, "Tachyon matter in boundary string field theory," JHEP **0207**, 025 (2002) [arXiv:hep-th/0205085].

[30] J. A. Minahan, "Rolling the tachyon in super BSFT," JHEP **0207**, 030 (2002) [arXiv:hep-th/0205098].

[31] A. Ishida and S. Uehara, "Gauge fields on tachyon matter," Phys. Lett. B **544**, 353 (2002) [arXiv:hep-th/0206102].

[32] K. Ohta and T. Yokono, "Gravitational approach to tachyon matter," Phys. Rev. D **66**, 125009 (2002) [arXiv:hep-th/0207004].

[33] N. D. Lambert and I. Sachs, "Tachyon dynamics and the effective action approximation," Phys. Rev. D **67**, 026005 (2003) [arXiv:hep-th/0208217].

[34] G. Gibbons, K. Hashimoto and P. Yi, "Tachyon condensates, Carrollian contraction of Lorentz group, and fundamental strings," JHEP **0209**, 061 (2002) [arXiv:hep-th/0209034].

[35] C. j. Kim, H. B. Kim, Y. b. Kim and O. K. Kwon, "Electromagnetic string fluid in rolling tachyon," JHEP **0303**, 008 (2003) [arXiv:hep-th/0301076].

[36] C. j. Kim, H. B. Kim, Y. b. Kim and O. K. Kwon, "Cosmology of rolling tachyon," arXiv:hep-th/0301142.

[37] F. Leblond and A. W. Peet, "SD-brane gravity fields and rolling tachyons," arXiv:hep-th/0303035.

[38] N. Lambert, H. Liu and J. Maldacena, "Closed strings from decaying D-branes," arXiv:hep-th/0303139.

[39] M. R. Garousi, "Off-shell extension of S-matrix elements and tachyonic effective actions," arXiv:hep-th/0303239.

[40] K. Okuyama, "Wess-Zumino term in tachyon effective action," JHEP **0305**, 005 (2003) [arXiv:hep-th/0304108].

[41] M. R. Garousi, "Slowly varying tachyon and tachyon potential," JHEP **0305**, 058 (2003) [arXiv:hep-th/0304145].

[42] C. j. Kim, Y. b. Kim and C. O. Lee, "Tachyon kinks," JHEP **0305**, 020 (2003) [arXiv:hep-th/0304180].

[43] P. Brax, J. Mourad and D. A. Steer, "Tachyon kinks on non BPS D-branes," Phys. Lett. B **575**, 115 (2003) [arXiv:hep-th/0304197].

[44] A. Sen, "Open and closed strings from unstable D-branes," Phys. Rev. D **68**, 106003 (2003) [arXiv:hep-th/0305011].

[45] O. K. Kwon and P. Yi, "String fluid, tachyon matter, and domain walls," JHEP **0309**, 003 (2003) [arXiv:hep-th/0305229].

[46] J. Kluson, "Particle production on half S-brane," arXiv:hep-th/0306002.

[47] G. N. Felder, L. Kofman and A. Starobinsky, "Caustics in tachyon matter and other Born-Infeld scalars," JHEP **0209**, 026 (2002) [arXiv:hep-th/0208019].

[48] M. Berkooz, B. Craps, D. Kutasov and G. Rajesh, "Comments on cosmological singularities in string theory," arXiv:hep-th/0212215.

[49] A. Sen, "Dirac-Born-Infeld action on the tachyon kink and vortex," arXiv:hep-th/0303057.

[50] A. Sen, "Moduli space of unstable D-branes on a circle of critical radius," arXiv:hep-th/0312003.

[51] A. Fotopoulos and A. A. Tseytlin, "On open superstring partition function in inhomogeneous rolling tachyon background," JHEP **0312**, 025 (2003) [arXiv:hep-th/0310253].

[52] A. Strominger and T. Takayanagi, "Correlators in timelike bulk Liouville theory," arXiv:hep-th/0303221.